Multimedia books in the mathematical education of engineers

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Abstract

Using IT in mathematical education has been a slow revolution. Although calculating tools have to some extent been used for several years, they are still controversial among many teachers. The most recent application is the multimedia book with links to Internet sites. These books include hypermedia and hypertext and what is called interactive objects and allow an explorative and personal way of working. The book Hypermath developed by the author is described in this paper. It is expected that the use of multimedia books will be important in future education of engineers.

1. Introduction

I have been involved with information technology (IT) and mathematical education for about 15 years. During most of that time, I have believed that ”the next two years” would bring a change in how professors and students teach and learn through the use of computers. Yet I am still waiting for these dramatic improvements. We may talk about the ”slow revolution”. However, I still believe that the changes will be revolutionary within ”the next two years”!

The obstacles to improved teaching and learning with IT are many [4]: Limited access to equipment, software and support, fragmented institutional planning, poor communication inside the educational institutions, focusing on technology instead of teaching and learning, underestimation of the difficulty of adopting new habitual behaviours, lack of information about ”Good practice”, faculty reward system pay little attention to IT, expectations for products are too high, too soon, if everyone is behind, who is ahead?

2. To understand understanding

Mathematical education for engineers has two main goals:

- To train students in logical and abstract thinking
- To give students an ability to use mathematical knowledge in other fields than mathematics itself (applying mathematics).

Achieving in particular the last goal will require a high level of what we call *understanding*. We then have to deal with what understanding really means. To understand the student must be able to *connect* the new subjects with something that is *important* to the student in a wider context than the upcoming examination. This means to connect new subjects with established knowledge. Whether this new subject is important to others is not relevant. A subject will not be regarded as important only because someone tells them it is. The student has to *personify* the knowledge and make it ”his own”.

To let the student experience importance the teacher has to use *visualisations* and *metaphors* familiar to the student. It will then be important to establish good visualisations and metaphors for mathematical concepts. In primary school this is widely used but at some point the practice comes to an end and the focus will gradually be pure and scientific. The most typical effect of this is a very steep curve of forgetting. The mathematical concepts do not concern them. A student experiencing importance in connection with mathematical education is a *motivated* student. If not he will be an unmotivated student.

To establish good visualisations and metaphors it will be necessary to use applications of mathematics connected to familiar situations. In many cases this means examples from daily life. The mathematical models may be defined not only by a set of equations but also by using pictures, animations and sound. The solution of the equations may easily be done by using a package on a computer so this is not the main problem. The main problem is connected to understanding in the sense we have recently talked about. This kind of understanding is then connected both to the mathematical concepts as such and to the application of them.

3. Using computer programs in mathematical education

Computer programs for symbolic calculation have been available for many years. The solution of standard mathematical problems in the engineering curricula, are now ”push button” problems. However this does not mean that teaching mathematics is old-fashioned. It’s still important to understand the mathematical concepts involved.

As an example let us look at finding the derivative. By using a computer program you only need to type the problem and push a button to solve it. The mathematical concepts involved may be a composite function, an outer function, a kernel, a substitution and a chain rule. The big question is then: Is the understanding of these mathematical concepts superfluous [5]? The solution of the problem may be carried out without knowing anything about them. We have to ask ourselves: Is the main goal of teaching mathematics to find the solution to a given problem? If the answer is yes, then we hardly need to give mathematical lessons at all. In my opinion this is a wrong conclusion for two main reasons.

- The formulation of a mathematical problem – to efficiently set up a model, which cannot be done by a computer, we often need to know about functions, variables, compositions and substitution of new variables (i.e. setting up the model in such a way that even a computer can solve it). It will not be possible to set up a mathematical model without knowing about and understanding the standard mathematical concepts.
- Access to the mathematical literature – this requires a firm understanding of the basic mathematical concepts.

The introduction of mathematical computing will no doubt have a significant impact on the education of mathematics. The largest effect of this change will be to concentrate on the concepts and on the formulation of models, and to skip lengthy calculations that may be solved by the computer.

4. Multimedia books

Multimedia books are computer documents showing text, graphics, sound, video and animation. These documents include what is called *hypertext* and *hypermedia*. Hypertext refers to words (or even pictures) in the document with a link to other documents or pages in the same document. Images, digital video and sound are similarly called hypermedia. By combining these elements we get applications that are [1]:

- *efficient* – you may replace information you read in a textbook with information you can see and hear.
- *direct* – you may get information using the best medium, such as a mathematics-teaching application.
- *personal and interactive* – you may control the flow of information and look for information in a non-linear or an event-driven manner. The information accessed depends on individual choices and will not be constrained to sequential turning of pages.

Multimedia gives us a new type of learning applications compared to the calculating packages: The electronic textbook. The electronic textbook may be distributed either on a CD-ROM or via the Internet. Compared to a standard textbook one may illustrate *processes* rather than *stationary conditions* and the calculating tool may be an integrated part of the book. Continuous mathematics often deals with time or space dependent phenomena and is not very well described in a standard textbook with only still-pictures.

One of the most interesting facilities using multimedia books is connected to understanding as described in section 2. By the use of multimedia visualisations are very well taken care of. Multimedia may in fact give us a virtual world or a *virtual reality*. Instead of only looking at a formula describing some space dependent phenomenon the student may move around interactively in a virtual space. Only the student's fantasy will be the limit.

The use of electronic books will encourage an *explorative* and interactive way of working. The book is event-driven and different students may look for different kinds of information depending on the specific needs of the student. The student may have a self-controlled dialog with the book. Interactive, event-driven books are in this way personified information and the student can adapt the use of the book to his own sphere and mathematical knowledge.

5. Hypermath, an interactive CD-ROM based mathematical learning system

Hypermath is a mathematical book with limited use of hypermedia. In addition to hypertext the use of graphics and animated graphics is implemented. The book covers basic functional analysis in the engineering curriculum and is divided into three parts: Functions, the derivative and integration. The start of each chapter gives a brief history of the subject with links to Internet sites.

A typical page in the book is composed of seven elements:

1. A brief introduction to the page subject
2. Vertical navigation buttons
3. Scrolling fields with solved problems

4. A horizontal bar including page turn buttons, a second window button, an intrinsic calculator and links to the different pages of the chapter (drop-down list)
5. *Exploring projects* - A button to interactively explore the page subjects.
6. Blue underlined words and some graphical objects (hypertext) with links to other pages or Internet sites.
7. Some pages may have a button to access *control projects,* which refer to a guided tour to examine a specific subject.

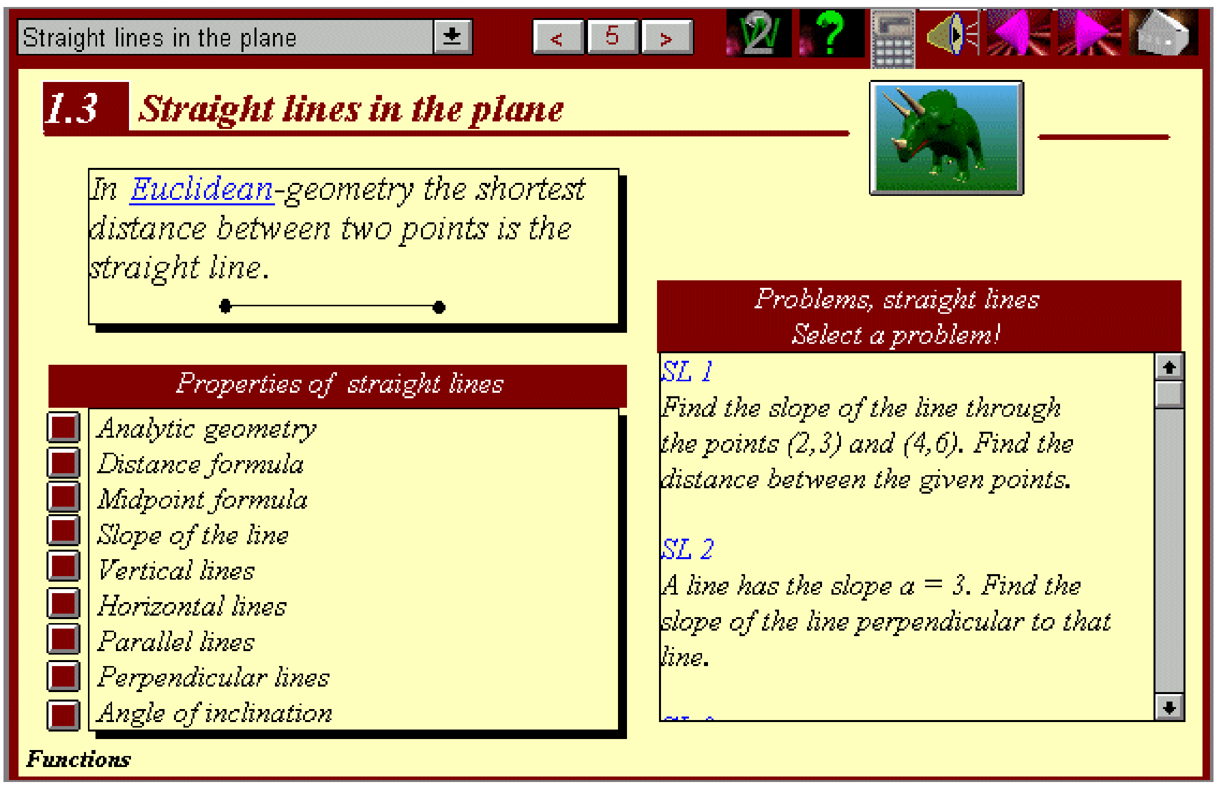
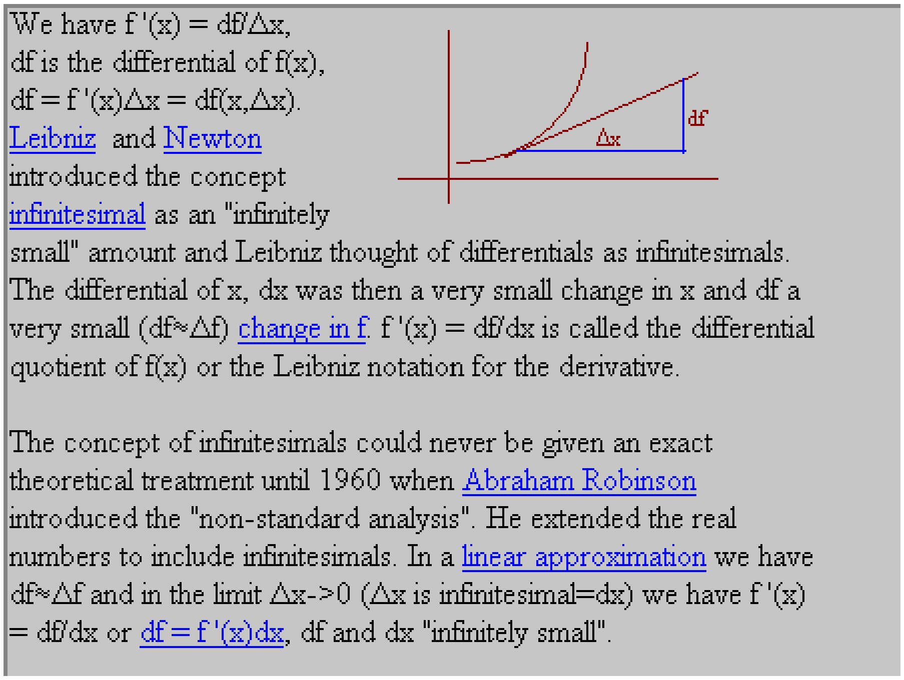


Fig. 1 Typical page of the book

The *interactive objects* of the book are :

- Hypertext – non-linear or event-driven control of the flow of information.
- Animated graphics – an example would be to actually see the development of a population by letting the number of individuals grow using animated graphics.
- Different kinds of buttons and boxes– navigation buttons (hyperlinks), radio buttons (selecting options) and drop-down lists (selecting options), for example multiple choice questions promptly responding.
- Pop up windows for different kinds of user-controlled actions, for example drawing lines by using the mouse and then automatically calculating the slope of the line.

Hypertext is the most common interactive object in multimedia documents. A non-linear flow of information may be demonstrated using the definition of the derivative in Hypermath. On this page the slope of the tangent is given by a formula. This formula is blue and underlined indicating hypertext. Clicking the formula leads us to another page. As we can see, 7 words are underlined. If we click on Leibniz a picture of Leibniz and a biography will appear in a separate window and similarly for Newton. If we click on infinitesimal the new page showing up has two underlined words. Clicking the word linear approximation leads us to a fourth page and clicking on Taylor series leads us to a fifth page. Different students may go in different directions depending on what they are looking for.



We have f '(x) = df/Δx,
df is the differential of f(x),
df = f '(x)Δx = df(x,Δx).
Leibniz and Newton introduced the concept infinitesimal as an "infinitely small" amount and Leibniz thought of differentials as infinitesimals. The differential of x, dx was then a very small change in x and df a very small (df≈Δf) change in f. f '(x) = df/dx is called the differential quotient of f(x) or the Leibniz notation for the derivative.

The concept of infinitesimals could never be given an exact theoretical treatment until 1960 when Abraham Robinson introduced the "non-standard analysis". He extended the real numbers to include infinitesimals. In a linear approximation we have df≈Δf and in the limit Δx->0 (Δx is infinitesimal=dx) we have f '(x) = df/dx or df = f '(x)dx, df and dx "infinitely small".

Fig 2 Hypertext words are underlined and blue

One of the obvious pitfalls with all this clicking is to be ”lost in cyberspace”. How to get back where I started? Hypermath has solved this problem by using different windows for different pages in connection with hypertext. This means that the page where you started is always in the background and may be moved to the foreground with a single click. In addition there are ”back buttons” as we know from the Internet and even ”history buttons” so that a specific page, viewed earlier, may be accessed by selecting it. After all, the *navigation problem* is one of the most challenging problems in connection with multimedia documents.

It is the interactive objects that make the difference compared to standard textbooks. As mentioned in section 4, this will give the student the possibility to explore subjects in his own way. The use of hypertext as recently described is a good example. Two different examples will be the use of the “explore” button to continuously (i.e. with small increments) change a functional parameter and the rotated angle of a surface of revolution. By moving the sliders in both directions the parameter and the angle change.

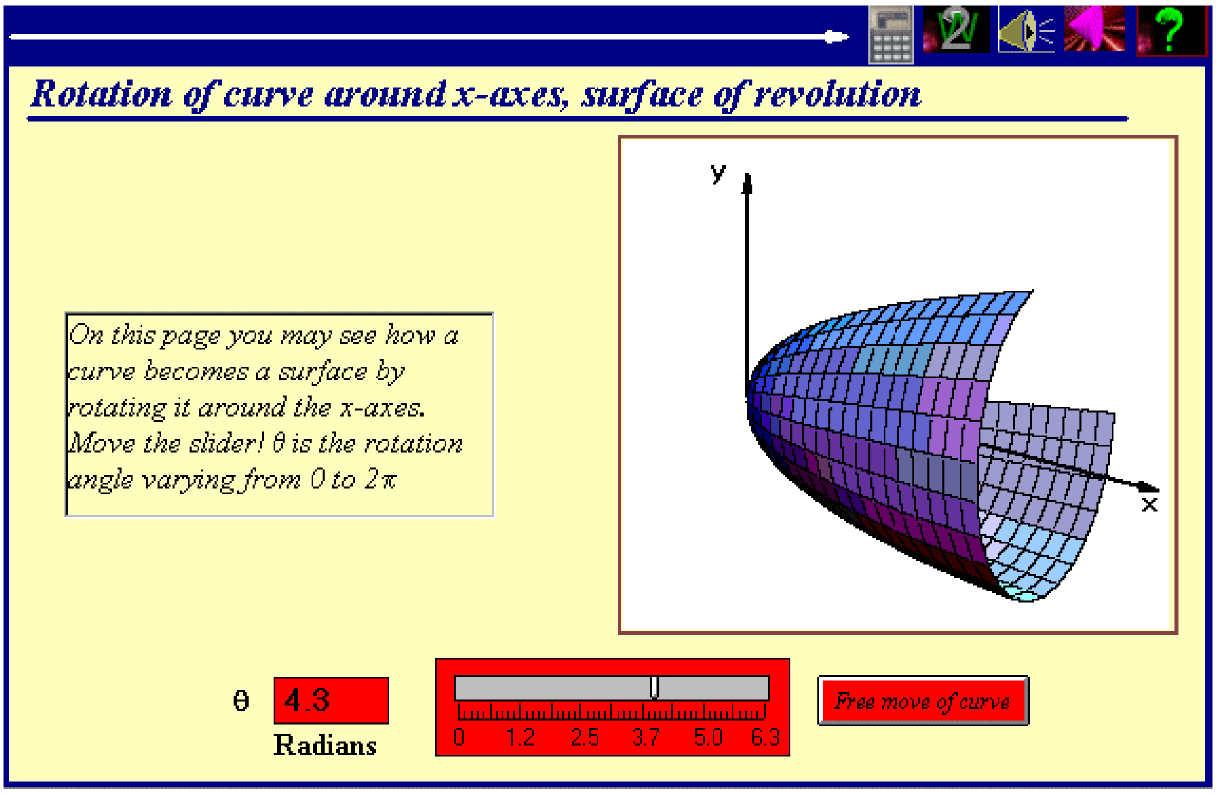


Fig 3 Surface of revolution

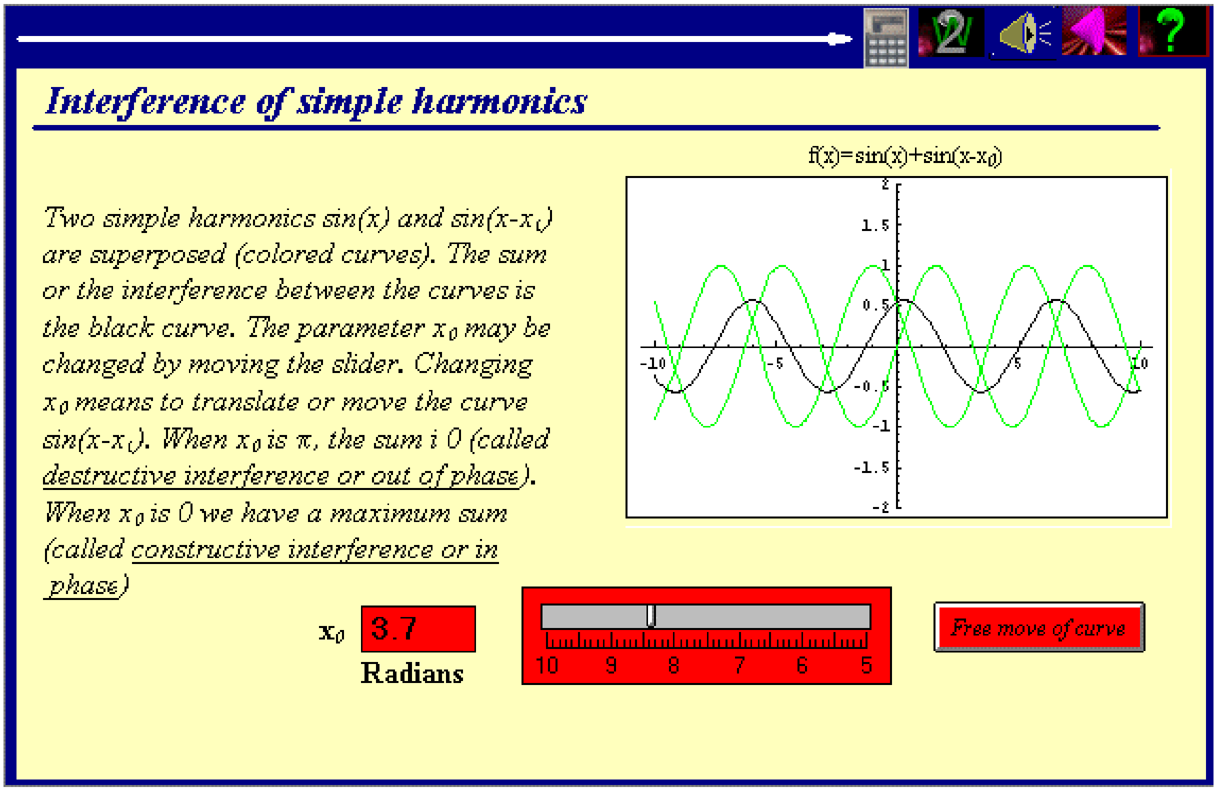


Fig. 4 Interference of curves

As mentioned in section 4 the illustration of processes rather than only stationary conditions is one of the most interesting features of the hypermedia book. This is widely used in Hypermath. Let us look at the development of a population of fish. The parameters in the fish population model are the death and birth rates. They may be controlled separately or both together. The visualisation of these changes is both a curve and animated graphics (fish appear or disappear at the chosen rate).

We may use this population model as an example of a control project. The student may click the mouse and a number of fish appear. The number of fish and the time parameter are inserted automatically in a table, in all seven pairs of numbers with seven mouse-clicks. By pushing another button a curve will be drawn. The student may start all over with a new choice of parameters, both the value of the parameter and the type of the parameter.

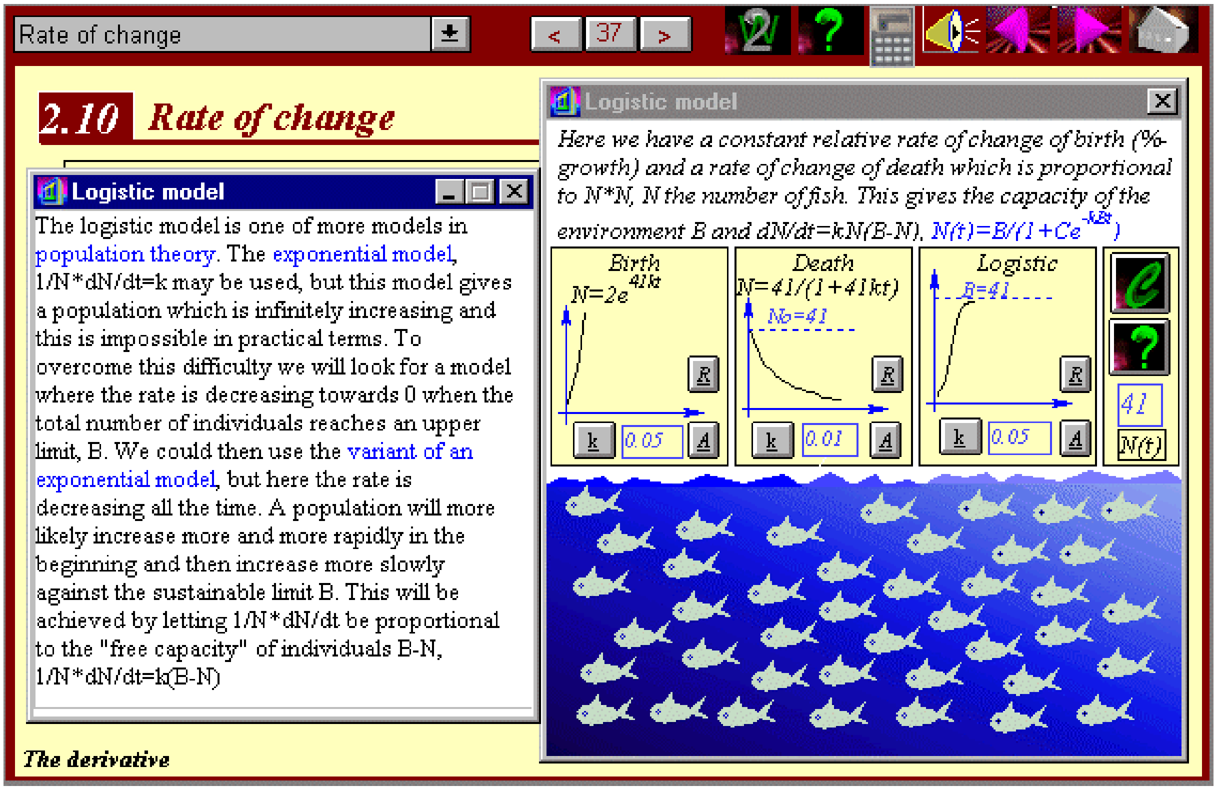


Fig. 5 Simulating a population of fish

The examples just mentioned are examples illustrating the application of mathematics. Many of the interactive objects visualise the mathematical concepts themselves. Two examples will be the *moving secant* becoming a tangent to illustrate the definition of the derivative and the sailing ship illustrating the mean value theorem. The secant may be chosen to move fast or slowly and the sailing ship may advance either using the automatic button or the step by step button.

The moving secant animation introduces one of the key problems in continuous mathematics: the understanding of the limit process. The solution to this problem is to replace the continuous process with a discrete process that is easier to handle for the human mind. Using animated graphics with the moving secant is an example, but another approach is to simulate the numeric values of limits by using a finite step numerical process. This is possible in Hypermath both for given functions and self defined functions. The user gives a value for the free variable and Hypermath calculates the functional change and the slope of the secant that approximates the value of the derivative for small steps.

Although computer programs may do your calculations, it is still necessary to have some knowledge of the underlying mathematical calculating algorithms. Traditionally the main effort has been on this kind of problem. In Hypermath the standard algorithms are implemented. By selecting an algorithm the student may use either the step by step button or the automatic button. Using the step by step button gives the reader perfect control over the development of the calculation, both forwards and backwards.

The book has a built-in numeric and graphics calculator. In addition to using it as a calculating tool it may also be used as a ”pedagogical” tool. Many of the exploring projects use the calculator to explore different subjects. As an example let us look at Riemann sums (integration theory). The student may approximate the definite integral by choosing different numbers of rectangles and different types of rectangles (left, right and midpoint) and in this way interactively explore the subject of Riemann sums.

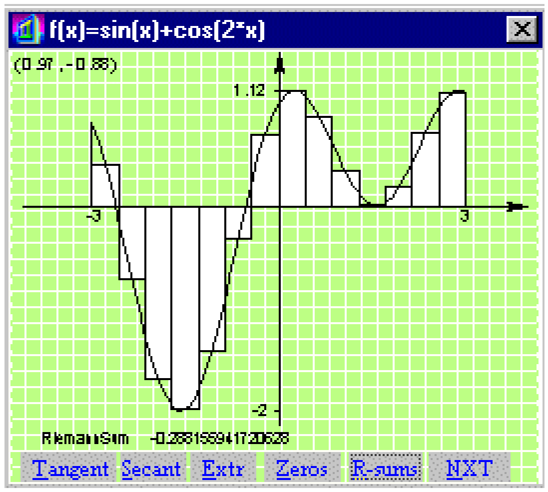


Fig 6 Riemann sums

Another example is the *moving tangent* to visualise the value of the derivative. An exploring project lets the student move the tangent and observe the extrema (slope=0) and points where the derivative is not defined (corner points). A control project on this page uses the built-in calculator to simultaneously show the curve of a function and the curve of the derivative of the function. The student will then see that the zero points of the derivative are the extrema of the function. One of the pages in a pop up window contains the word ”successive zooms” which is underlined and blue, indicating hypertext. Clicking it will give the student a possibility to refine the extrema .

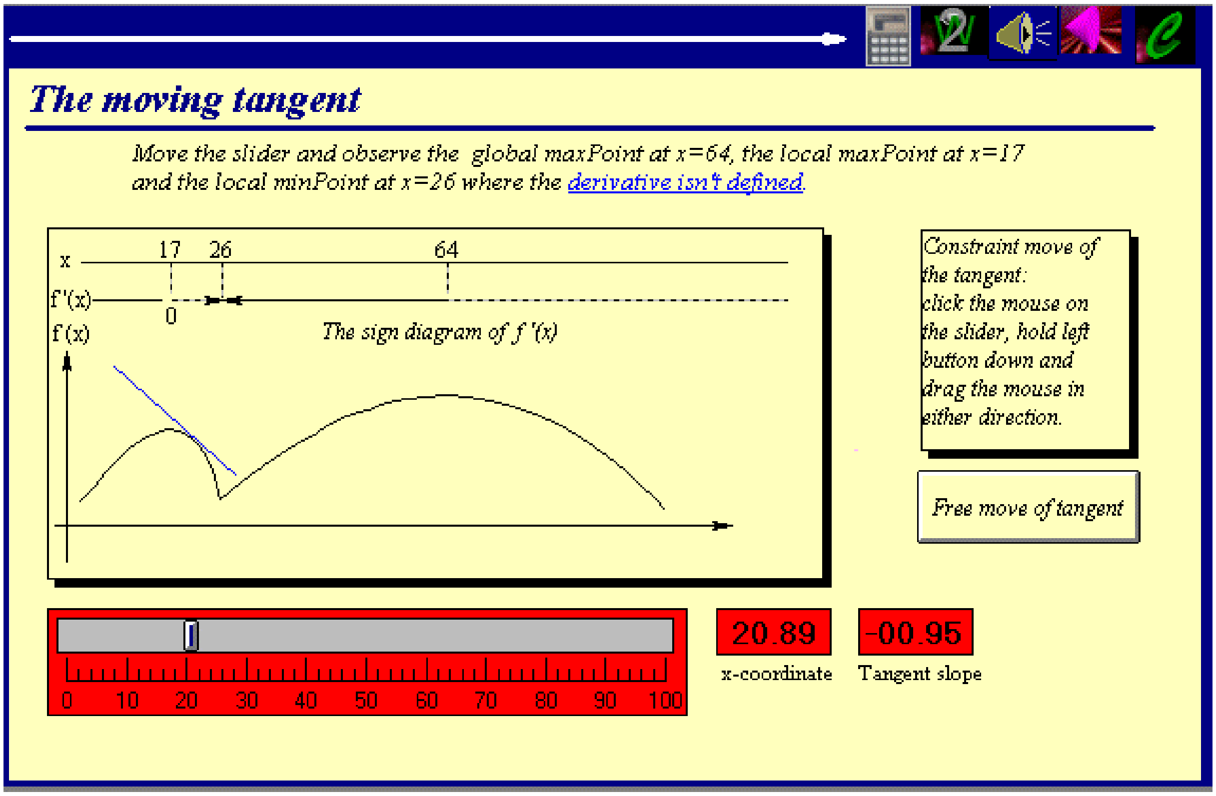


Fig. 7 The moving tangent project

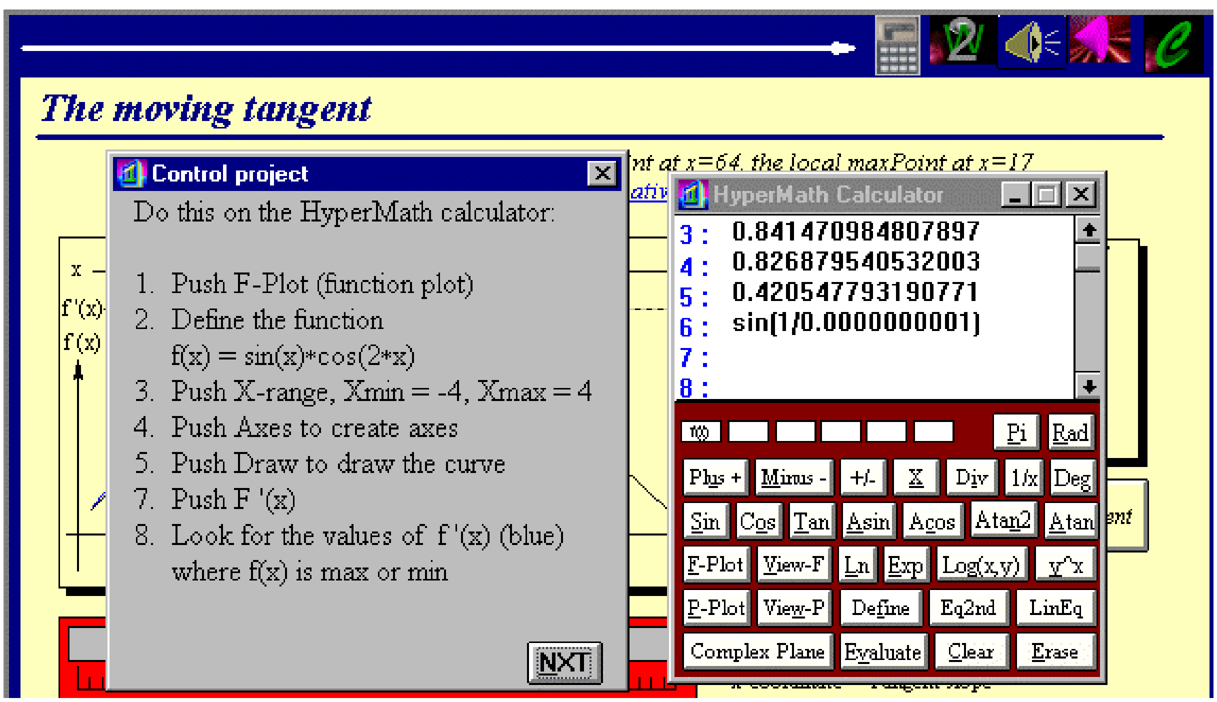


Fig 7 Extrema control project

Let us now consider what the student has done. Rather than only looking at the formula for the derivative function and exact values for the extrema (which may be found by pushing a button in a symbolic system), he has been involved in a *self-controlled process*. Thus the student has had much better possibility to understand the subject of differentiation in the sense we have earlier defined in section 2. Traditionally, the need to master the calculating algorithm has lead to a striking confusion between the algorithm and the underlying concept. Mastering is not always the same as understanding!

As a last example of using the built-in calculator, I want to show a calculation on complex numbers. To calculate the sum of two numbers you are not only given the numeric answer, but also a visualisation of the complex numbers (”arrows”) involved.

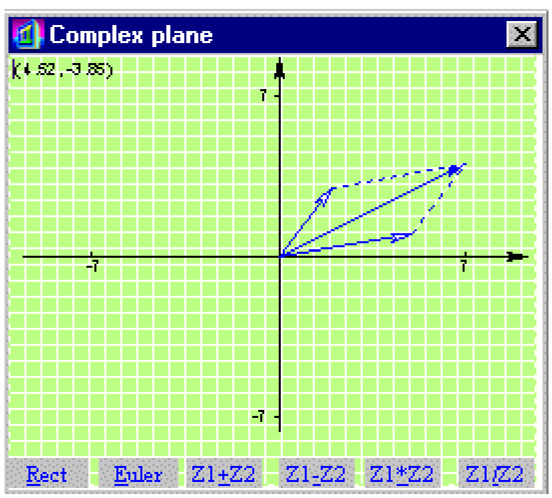


Fig 8 Adding complex numbers

Hyperlinks to symbolic mathematical systems are implemented. When a student is looking for solved problems, he may define his own problem and then use the hyperlink to solve it using the selected system.

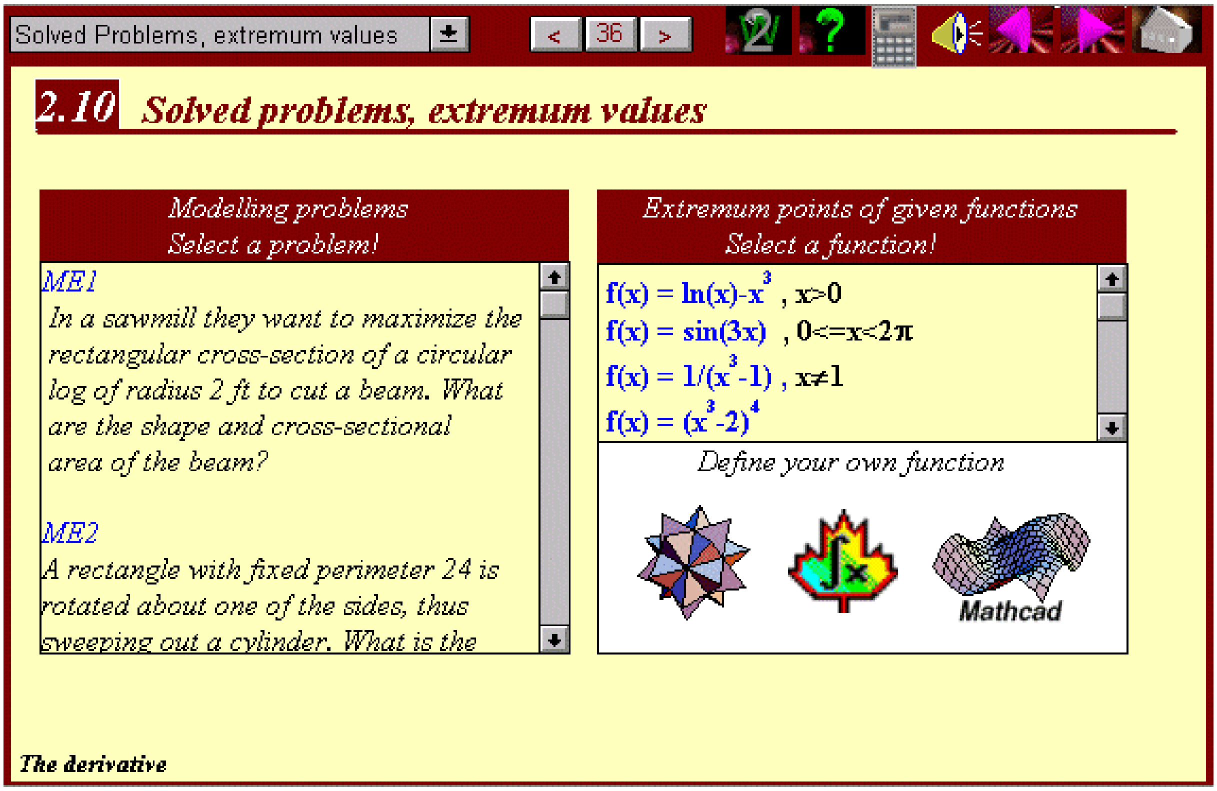


Fig 9 Hyperlinks to symbolic applications

Using CD-ROM or the Internet a huge number of solved problems (in detail) can be stored. An ordinary textbook must have thousands of pages to match this. The navigation through such a huge number of problems has to be event-driven and not page sequential so the multimedia book is the only feasible medium.

To enhance the use of the book it is possible to view two pages simultaneously, the main page and the second page. The second page appears in a separate window and may be controlled independently of the main page. While looking at the definition of the derivative on the main page the student may look at the application of it in the second window.

6. Conclusion

The use of multimedia books distributed either on CD-ROM or via the Internet will be important in future education. It will enhance the understanding of mathematical concepts and the application of mathematics.

Distribution via the Internet or as combination of the net and a CD-ROM will make possible a form of education that is independent of time and place. The development of information- and communication technology will give us new possibilities within education. In addition to concepts like ”distance learning” and ”education by letters” we have got new concepts like ”open learning”, ”virtual schools” , ”virtual classrooms” , ”the digital university” and ”distributed teaching” .

What will be the role of the professors in this digital world? At the moment there is too little experience to give definitive answers to that question. In my opinion the need for professors in education will not be less important. After all, teaching is a social process between human beings.

REFERENCES:

[1] ASYMETRIX CORPORATION (1994): Multimedia toolbook, *User manual*

[2] O. BRINGSLID (under construction) Report on Hypermath, *Research Council of Norway*

[3] O. BRINGSLID (1998): Hypermath, an electronic textbook in engineering mathematics, *Proceedings of 9th SEFI seminar on mathematics*

[4] S. GILBERT (1996): Making the most of a slow revolution, *Change,* Mars, April pp. 10

[5] F. SIMONS (1998): Computer algebra and the service teaching of mathematics, *Proceedings of 9th SEFI seminar on mathematics*