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Abstract

Interactive mathematics using mathematical notation on the web have up to now not been possible. The development of the XML-standard MathML and also the use of JAVA servlet containers have made it possible to use the web as an advanced calculator using numeric, graphic and symbolic mathematics interactively. In connection with mathematical e-learning for engineering students this will represent quit a quantum step. In many European countries both the number of students in mathematical courses generally and the number passing through the exams has reached a “dangerous” low level and industry and schools are complaining. To meet this challenge the pedagogics and the methods of communicating mathematics will have to undergo an innovative process where new technology is important. The improvement of understanding mathematics by using interactive and personalized documents on the web could turn bad trends and the problems associated with girls and mathematics also have to be emphasized trough the selection of applications.

1. Introduction

It is expected that the use of multimedia books and the Internet will be important in future mathematical education of engineers [1]. Until now the use of web sites have been of less importance because of the limitations in the standard mark-up HTML [3]. Rendering mathematical symbols and operators in a web browser have in fact not been possible otherwise than using images where especially the semantics of mathematical elements are missing.

Encoding mathematics for computer processing is much older than the Web. The use of T_EX [2], was already in wide use before the Web. Better solutions are required for handling problems like on-screen working and interconnectivity and the common mark-up standard HTML has a lot of limitations. The use of image-based methods (*.gif,*.jpg) are not adequate.

It is believed that more sophisticated mark-up languages will be important in the education of mathematics using the Web. This is not merely a display problem. The Web represents a fundamental change for knowledge storage where the reuse of mathematical content and ways of communicating mathematical insight and understanding will play a central role. Combined with the ability of using online calculations and the use of animated graphics could turn the Web to an excellent professor.

2. MathML [4]

The traditional mark-up language on the Web is called HTML. HTML has, in connection with mathematics, many limitations. Document quality is poor, authoring

is difficult, and mathematical information contained in images is not available for searching and reuse in other applications.

To solve these problems W3C [6] has developed an application of XML [5] called MathML. XML stands for eXtensible Markup Language which means that it may be used to develop applications designed for different areas using appropriate extensions. MathML (Mathematical Markup Language) includes tags both for presentation and content (semantics) connected to mathematical symbols and operators. MathML is not intended for editing by hand, but is for handling by specialized software, or for export to and from suitable math packages.

This simple example of MathML gives you an idea of how it works [6]. The equation in question is: $\mathbf{x^2 + 4x + 4 = 0.}$ The presentational tags generally start with "m" and then use "o" for operator "i" for identifier "n" for number, and so on. The "mrow" tags indicate organization into horizontal groups. The semantic tags take into account such concepts as "times", "power of" and so on and may be understood as mathematical concepts and evaluated by mathematical software packages.

Presentation mark-up	Content mark-up

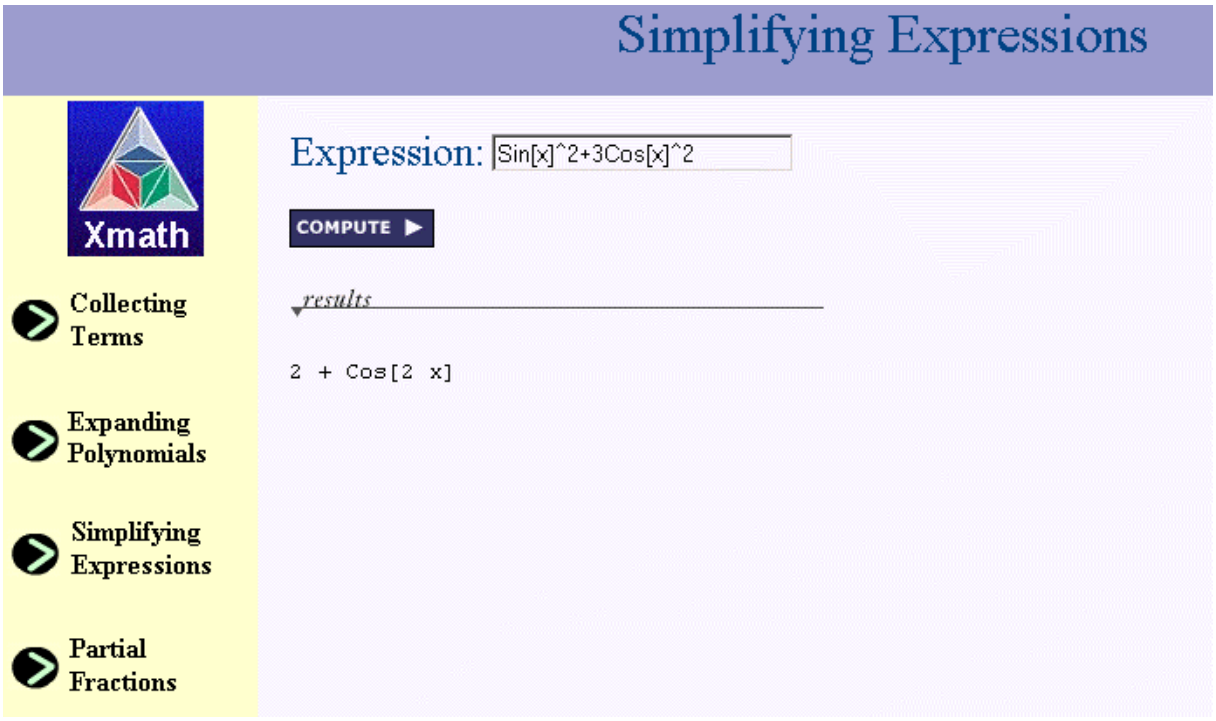
<mrow> <mrow> <msup> <mi>x</mi> <mn>2</mn> </msup> <mo>+</mo> <mrow> <mn>4</mn> <mo>⁢</mo> <mi>x</mi> </mrow> <mo>+</mo> <mn>4</mn> </mrow> <mo>=</mo> <mn>0</mn> </mrow>	<apply> <plus/> <apply> <power/> <ci>x</ci> <cn>2</cn> </apply> <apply> <times/> <cn>4</cn> <ci>x</ci> </apply> <cn>4</cn> </apply>

The equation using content mark-up may be pasted from the Web into mathematical software and solved. Using f.ex. Mathematica [7] one could then add mark-up for solving (……..<mi>Solve</mi>…….) and get the answer $x = -2$, $x = 2$ when evaluated by Mathematica. The Xmath team [8] will use Mathematica and the plug-in IBM Techexplorer [9] in connection with MathML (*.mml) files. The Internet Explorer cannot render *.mml-files directly.

3. Online calculations

The access to online calculations has been limited. Recently one of the leaders in mathematical computing has released the webMathematica [10] giving the user a possibility to connect to a server running *Mathematica* [7].

An example connected to the Xmath project [8] will typical look like:

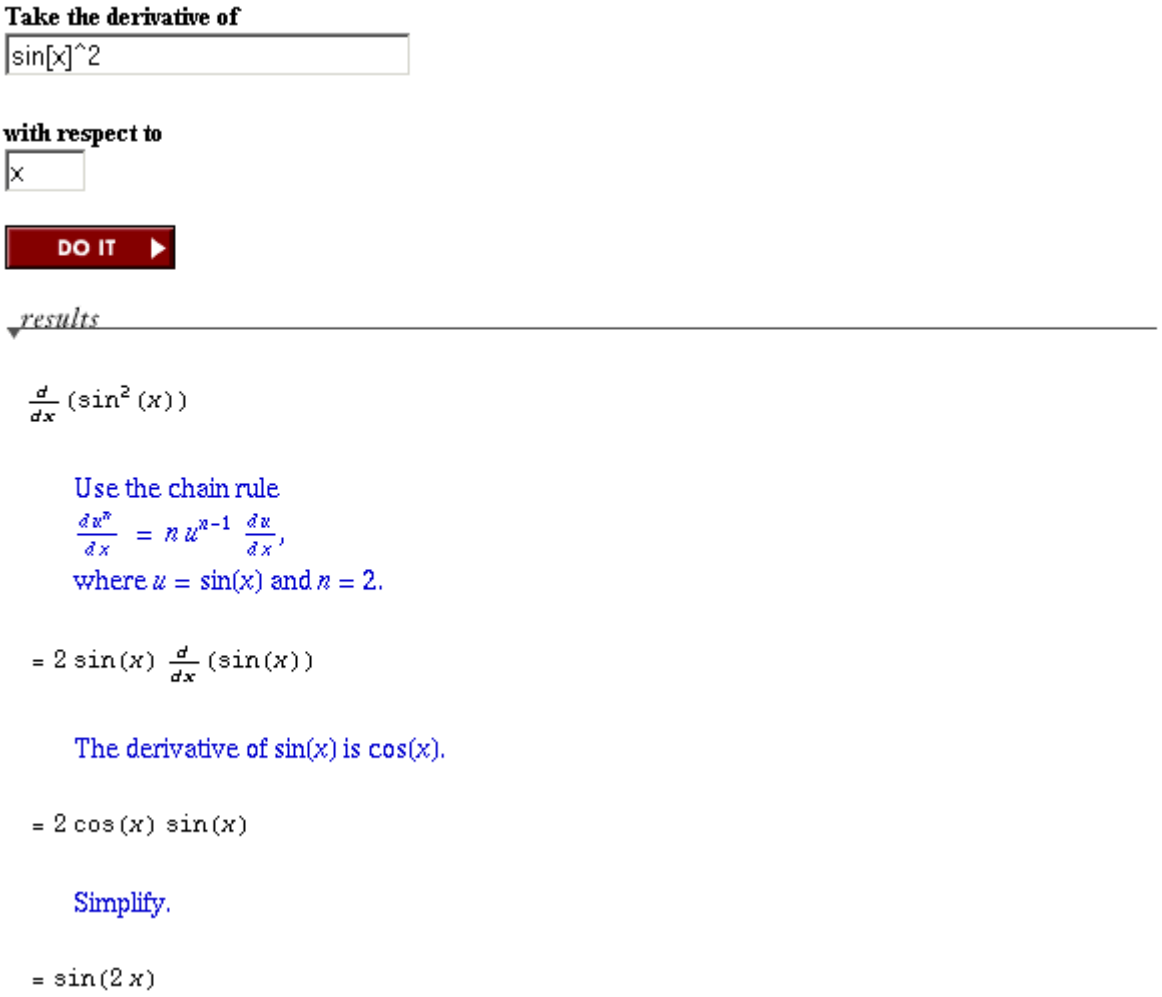


In the field “Expression” one may put in some expression to be simplified and a remote server containing *Mathematica* will evaluate it.

The source files are socalled *.msp files. MSP stands for *Mathematica* Server Pages. They are based on a standard Java technology called servlets. Servlets are special Java programs that run on a web server machine. Essentially all modern web servers support servlets natively or through a plug-in servlet container, f.ex Jrun [11] used by the Xmath team [8]. *J/Link* [12] integrates *Mathematica* and Java.

In this way one may add interactive calculations and visualization to a web site. These are important elements in a web-based course in mathematics and it also means that it will be possible to include step-by-step calculations directly on site. The students may try their own examples and both the step by step calculations and the

mathematical rules applied will show up [13].



MSP files may be generated by using a text editor (Notepad f.ex.). The special

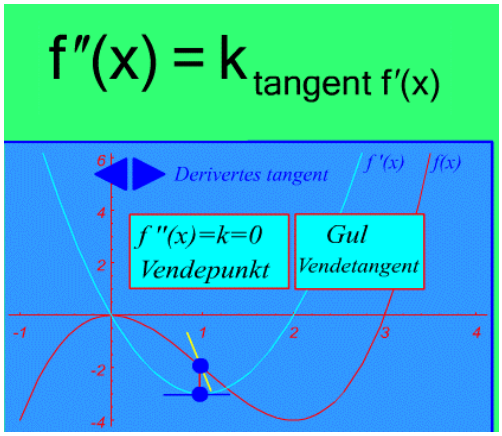
Mathlet commands have to be included in the HTML mark-up of the page:

```
<%Mathlet MSPValue[ $$expr,  "x+y"] %>
      <b>to the</b>
      <input type="TEXT" name="num" align="LEFT" size="3" value =
      <%Mathlet  MSPValue[ $$num, "4"] %>
      <b>power</b>.
      <p>
        <input  type="image"  name="submitButton"    img  src="Design/compute.gif"  width="83"
height="22" border="0" value="Compute">
      </p>
      <p> <img src="Design/results.gif" width="306" height="17"><br>
        <br>
        <%Mathlet
      MSPFunction[ {$$expr, $$num},
      MSPFormat[ Expand[$$expr^$$num], StandardForm]] %>
```

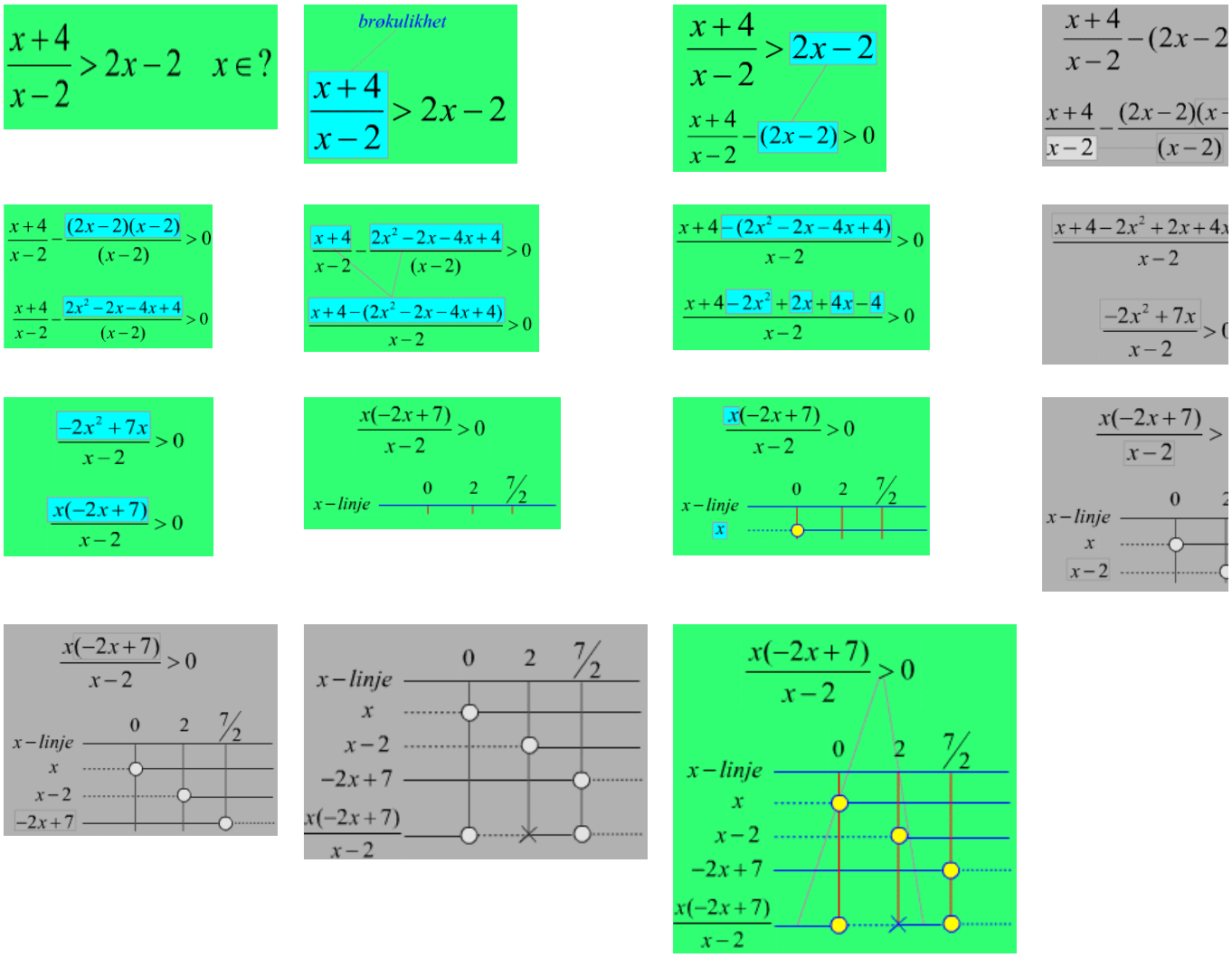
4. Flash animations [14]

One of the most distinctive qualities using computer mathematics is the use of animations to visualize mathematical concepts. On the Web the file size is decisive and it is important to use scalable master objects and instances. Macromedia Flash uses vector graphics with a minimum of bytes to download (low bandwidth). A symbol is a reusable image, animation, or button. An instance is an occurrence of a symbol on the Stage or nested inside another symbol. Symbols can make editing a movie (Flash file) simpler as changes to repeating elements need only be made to the symbol and Flash updates all instances.

An example from the Xmath project [8] is shown. Clicking the triangle buttons will show the user how tangents are changing and the yellow tangent is the tangent of inflexion where the derivative has a horizontal tangent.



Another example of Flash animations is a lesson with synchronized sound where the user may stop or reverse the player to look closer to one of the pictures or hear the lesson once again. The animations below follows from left to right .



5. ICT and mathematical education

The connection between mathematical pedagogics and ICT has to some extent been underestimated. Using the Web to maximise learning outcomes is one application of an information and communication technology that impacts on teaching practice. Online projects can bring experts into the classroom by using email, discussion groups, links to other websites and different kinds of communities focusing both on mathematics as a scientific subject and on the teaching and learning of mathematics. For many professors the use of online resources is a relatively new and exiting

method of meeting curriculum needs and delivering learning outcomes.

Mathematics and ICT is in its very birth. One very important factor of this shortcoming has been the difficulties with distributing mathematics on the Internet. Until now this has in fact not been possible due to technological problems and the lack of standards. Mathematics on the net has NOT been mathematics but images. The development of the new standard for describing documents on the net, XML (MathML), has now made it possible to include REAL mathematics on a web page and to introduce this brand-new technology in connenction with mathematical education. MathML will convert "frozen images" to "live materials", the main difference between ordinary books and multimedia documents. Introducing MathML will be an important strategic factor in the development and innovation of mathematical and scientific education on the net and represents a "quantum step" in rendering mathematics in a web browser.

The use of online resources are expected to enhance mathematical pedagogics and to stimulate activity, independence and collaboration. Online resources are multimedia documents which will give better understanding and allow an explorative way of working [1]. This may also prevent sharp distinctions between girls and boys using ICT and the problems with girls and mathematics have to be focused through the selection of applications .

In connection with distance learning special focus has to be put on the communication process which is one of the main differences between ordinary teaching and distance learning. The communication process is critical and

mathematical didactics has to deal with these kinds of problems. It is to hope that the use of MathML documents in mail and chat will enhance communication. The use of MathML will strongly support one of the most important parameters here: the exactness and unambiguousness of the communication. Innovative pedagogical methods in connection with virtual rooms for group activities using small groups will be important. Important differences between net-based pedagogics and traditionally pedagogics lies in this communication process and in the ability to monitor the students work . You then have to use a management system which may be called a vertical portal for online learning.

6. Conclusion

The overall profit of using interactive mathematics on the Web will be to stimulate the interest of mathematics. In the whole western world the students are missing in mathematical courses as well as in courses in natural sciences. UNESCO [15] has pointed this out very clearly and it will turn out to be a very serious problem. It is imperative to turn this development. Innovation in connection with the way we communicate mathematical courses will be a factor of strategic importance.

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[9] IBM Techexplorer http://www-4.ibm.com/software/network/techexplorer/

[10] Wolfram Research http://www.wolfram.com/products/webmathematica/

[11] Allaire http://www.jrun.com./

[12] Wolfram Research http://www.wolfram.com/solutions/mathlink/jlink/

[13] Wolfram Research http://library.wolfram.com/webMathematica/MSP/Explore/Education/WalkD

[14] Macromedia http://www.macromedia.com/software/flash/contents.html

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